Errors and Uncertainty

A value without uncertainty contains no information!

- As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." (Albert Einstein)
- "In this world, nothing is certain but death and taxes." (Benjamin Franklin)
- "As complexity rises, precise statements lose meaning, and meaningful statements lose precision." (Lotfi Zadeh)
- "The whole problem with the world is that fools and fanatics are always so certain of themselves, with wiser people so full of doubts." (Bertrand Russell)
- "We demand rigidly defined areas of doubt and uncertainty." (Vroomfondel - in "The Hitchhiker's Guide to the Galaxy" by Douglas Adams)

Sources of Error

- **Static**
  - Errors that are present when the input is constant.
- **Dynamic**
  - Errors that are present when the input changes.
- **Drift**
  - Errors that result from sensor changes with time.
- **Exposure**
  - Errors due to imperfect coupling between the sensor and what is measured.

Definition of Errors

The term *error* represents the imprecision and inaccuracy of a measurement or numerical computation.

$$\epsilon_{abs} = \left| (\tilde{\chi} - \chi) \right|$$

Absolute Error – The magnitude of the different in the **approximation ($\tilde{\chi}$)** and the **true value (χ)**.

$$\epsilon_{rel} = \frac{\left| (\tilde{\chi} - \chi) \right|}{\left| \chi \right|}$$

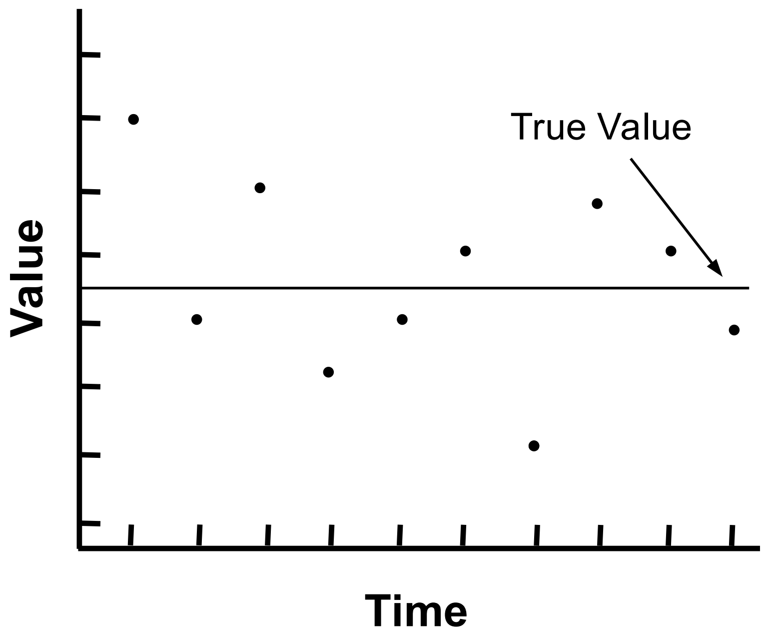
Relative Error – The absolute error normalized (relative) to the true value.

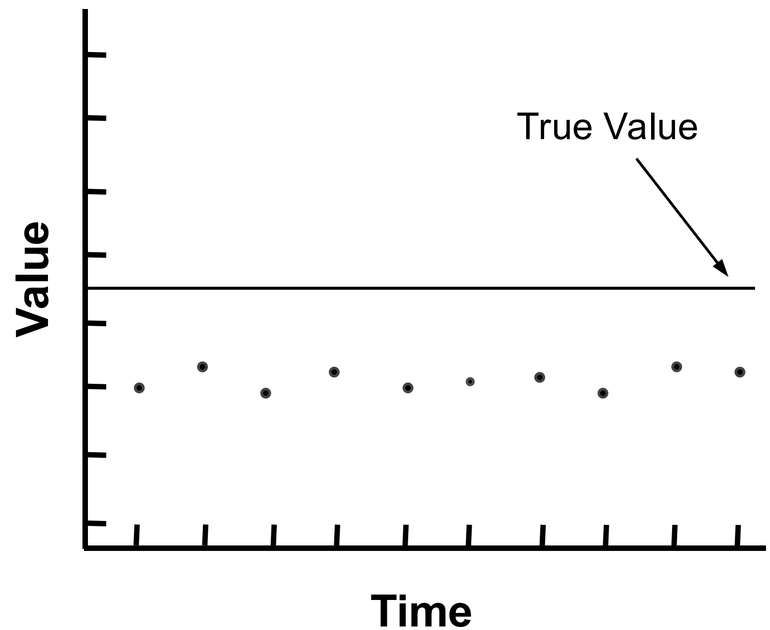
Random and Systematic Errors

- Measurement uncertainty is due to random and systematic errors.
- Random errors are statistical fluctuations in the measured data due to the precision limitations of the measurement device.
    - Random errors usually result from the experimenter's inability to take the same measurement in exactly the same way.
- Systematic errors are reproducible inaccuracies that are consistently in the same direction.
    - Systematic errors are often due to a problem which persists throughout the entire experiment.
- Mistakes made in calculations, in reading values, and in using instruments are not considered in error analysis.
- Typically, it is assumed that the experimenters are careful and competent; however, some experimenters are more careful than others so it is important to know the experimenters.

Accuracy and Precision

- Measurements and calculations can be characterized with regard to their accuracy and precision.
- Accuracy refers to how closely a value agrees with the true value.
- Precision refers to how closely values agree with each other.

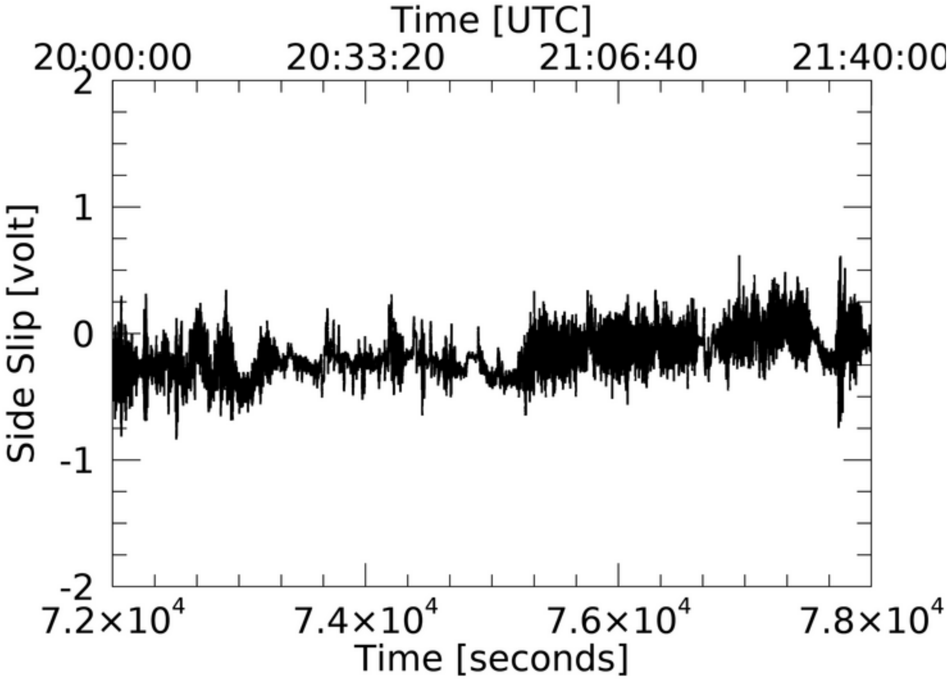




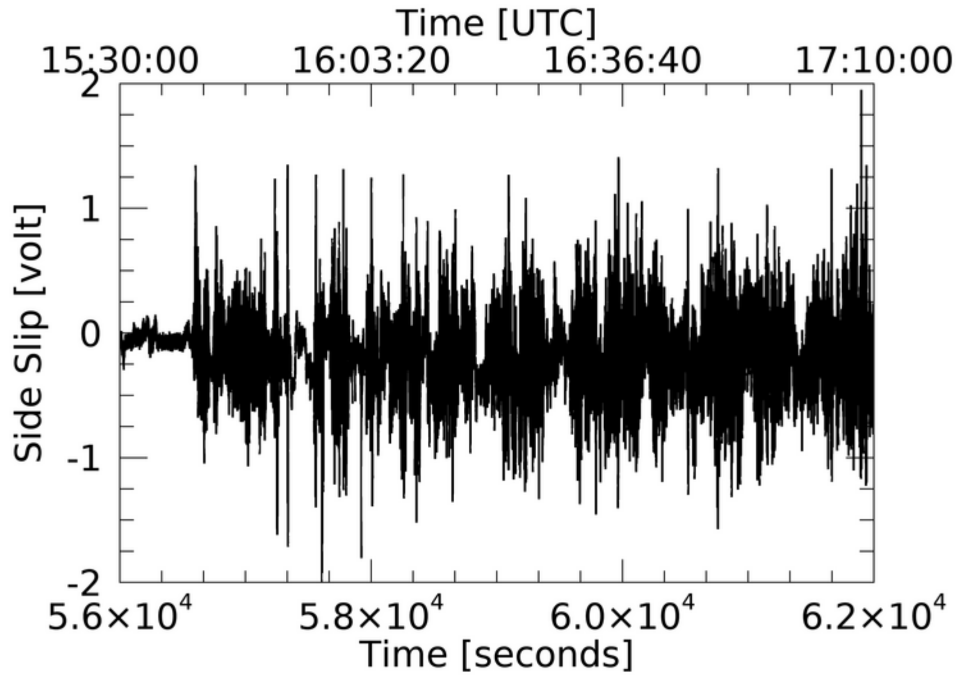
What does this indicate about repeatability?

Voltage Measurements (25 Hz)

29 SEP 2005



2 SEP 2012

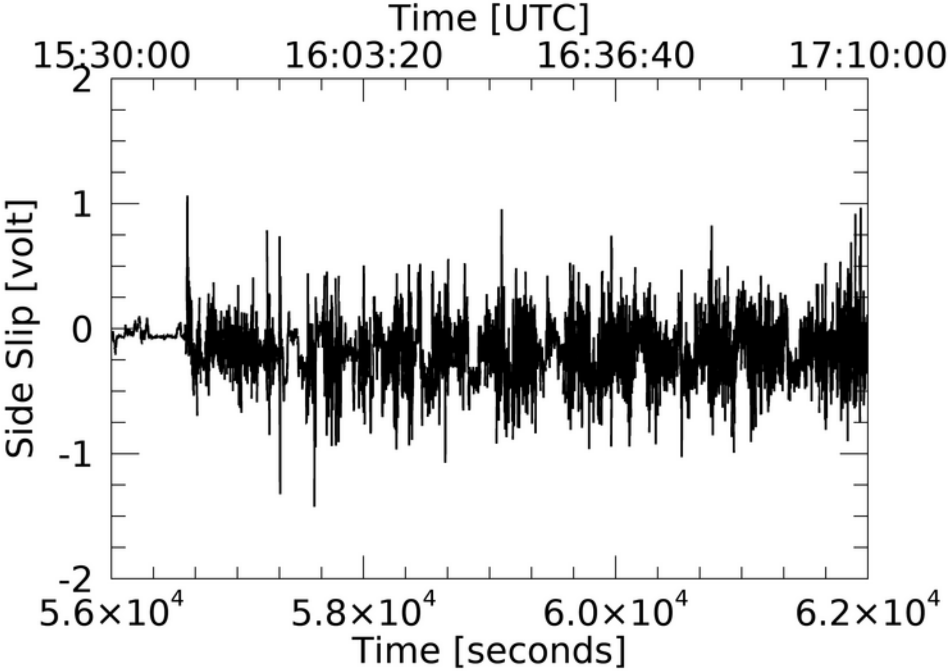


Time series (25 Hz) of voltage from the side slip pressure transducer on the North Dakota Citation Research Aircraft during research flights. The same data system and analog-to-digital board is used on both flights.

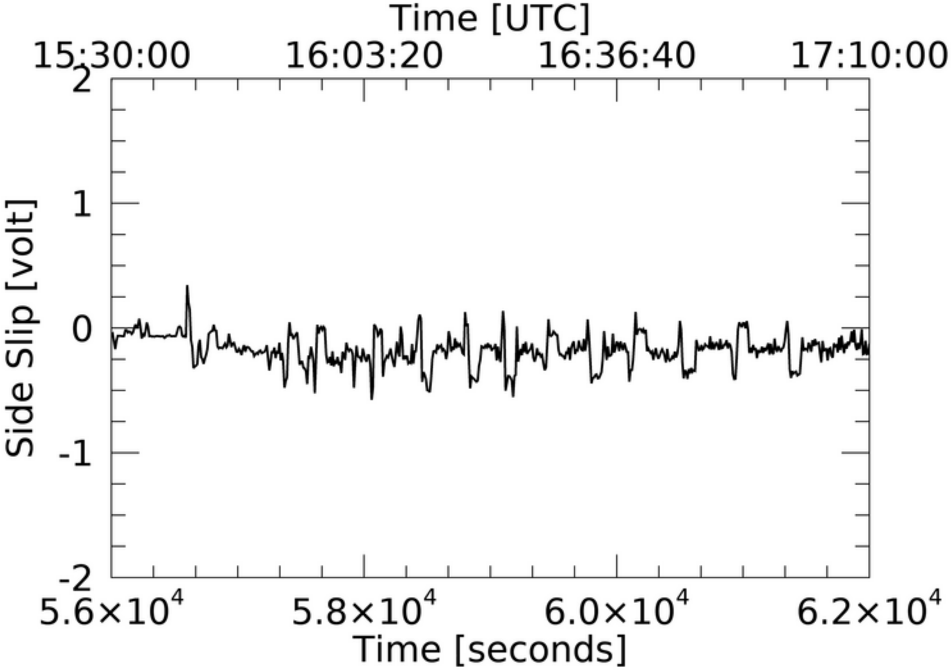
Uncertainty Improvements

- To improve our certainty in a measurement, we can make repeated measurement.

1 Hz (25 Points)



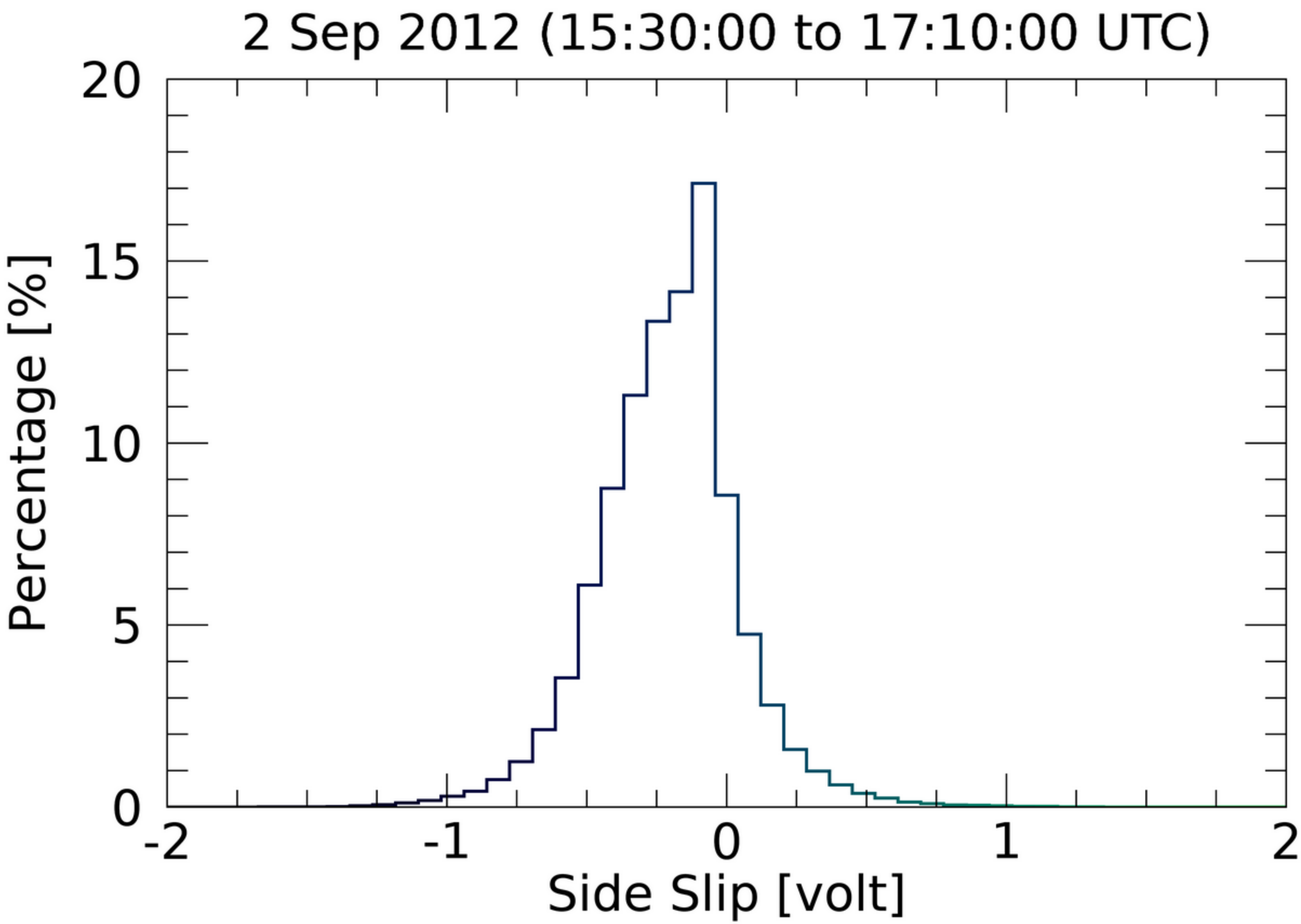
10 sec (250 Points)



Time series with different averaging of side slip pressure transducer voltage on the North Dakota Citation Research Aircraft during 2 September 2012 research flights.

Uncertainty Improvements

- Repeated measurements can be represented as a Histogram that represents how the measurements are distributed.



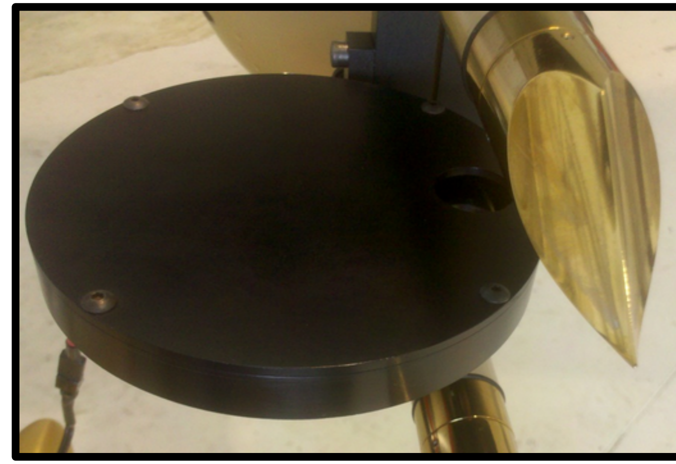
Statistical Characterization

- What single number best characterizes the complete group of measurements?
  - Mode – The value(s) the come up most often.
  - Median – The value where 50% of the measurements are above the value and 50% below.
  - Mean – Sum of all measurement divided by number of measurements.
- What single number best characterizes the Span, Spread and Symmetry?

Statistics of side slip pressure transducer voltage during 2 September 2012 Citation research flight (15:30-17:10 UTC).

Side_Slip [volt]	
X-Span	
Number of Points	150025.0000
Summation	-25805.9380
Minimum	-2.1022
Maximum	1.9507
X-Location	
Mean	-0.1720
Trimean	-0.1666
Median	-0.1567
X-Spread	
Sample Standard Deviation .	0.2577
Interquartile Range	0.2931
Mean Absolute Deviation ...	0.1931
Median Absolute Deviation .	0.1439
X-Symmetry	
Population Skewness	-0.0312
Sample Skewness Coefficient	-0.0312
Yule-Kendall Index	-0.1348
X-Percentiles/Quantiles	
0.05 Percentile	-0.5930
0.25 Percentile	-0.3230
0.50 Percentile	-0.1567
0.75 Percentile	-0.0299
0.95 Percentile	0.2237

Indicating Uncertainty



- **Instrument Calibration**
  - Instruments are calibrated,

    not data or data sets.
  - Provides the uncertainty by comparing instrument performance with a know standard, which is used for processing and data analysis.
- **Instrument Performance**
  - Confirm Instrument is working as expected.
- **Significant Figures**
  - All but the last number is certain.
  - The last number is uncertain.

What is the range of 20.8 ^{0}C?

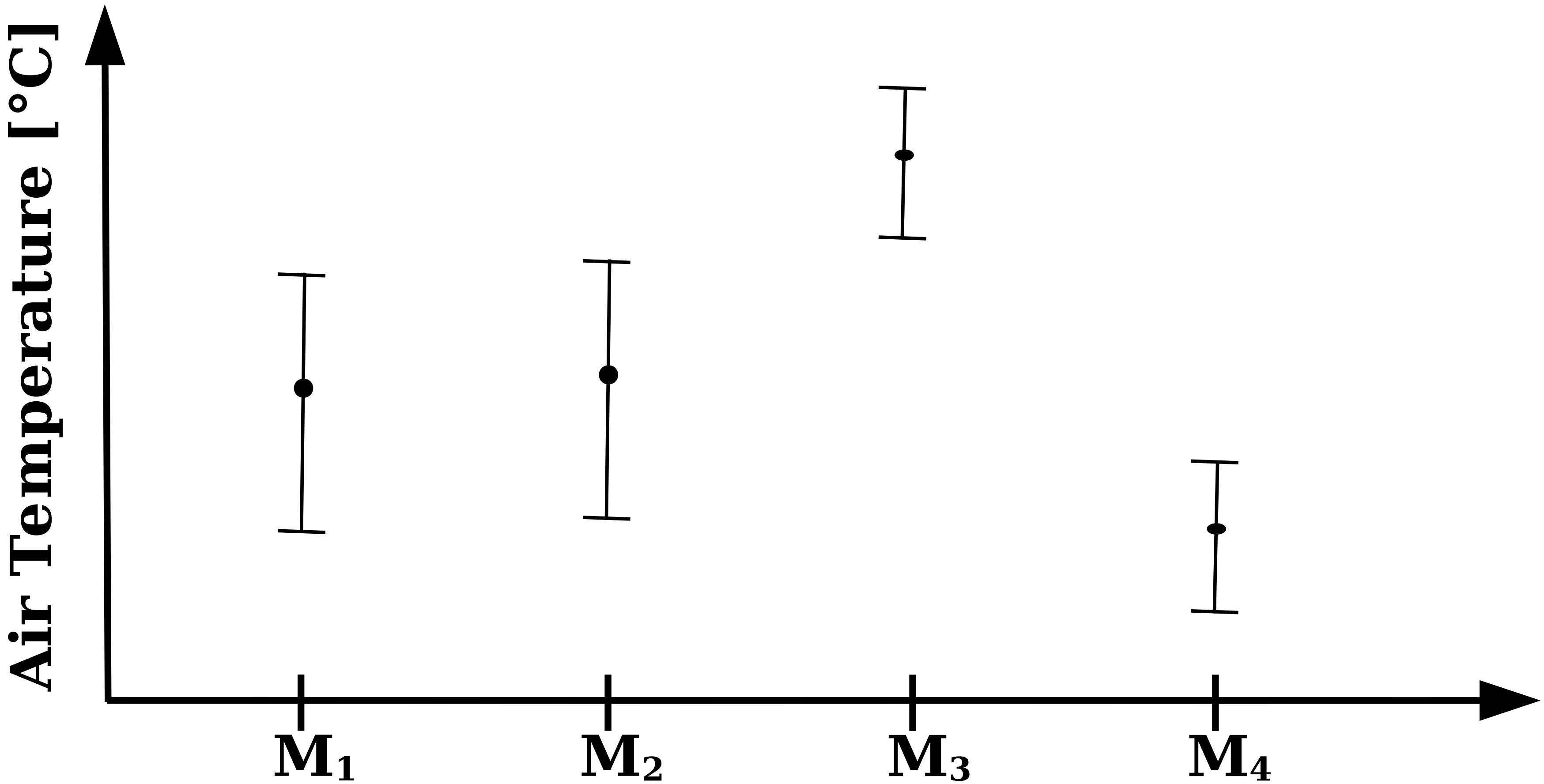
Calculated Quantities Uncertainty

- A single measurement rarely gives the information desired.
- Calculated quantities may contain several measurement each with an uncertainty and it is the uncertainty in the calculated quantities we are interested in.
- We could assume the errors in the measured quantities combine in such a way as to drive the value of calculated quantity as far as possible from the central value.
- It is more probable for the uncertainties in the basic measurement to combine in a less extreme matter so the calculated quantity error would be a probable uncertainty.

Total Temp. + TAS → Air Temp.

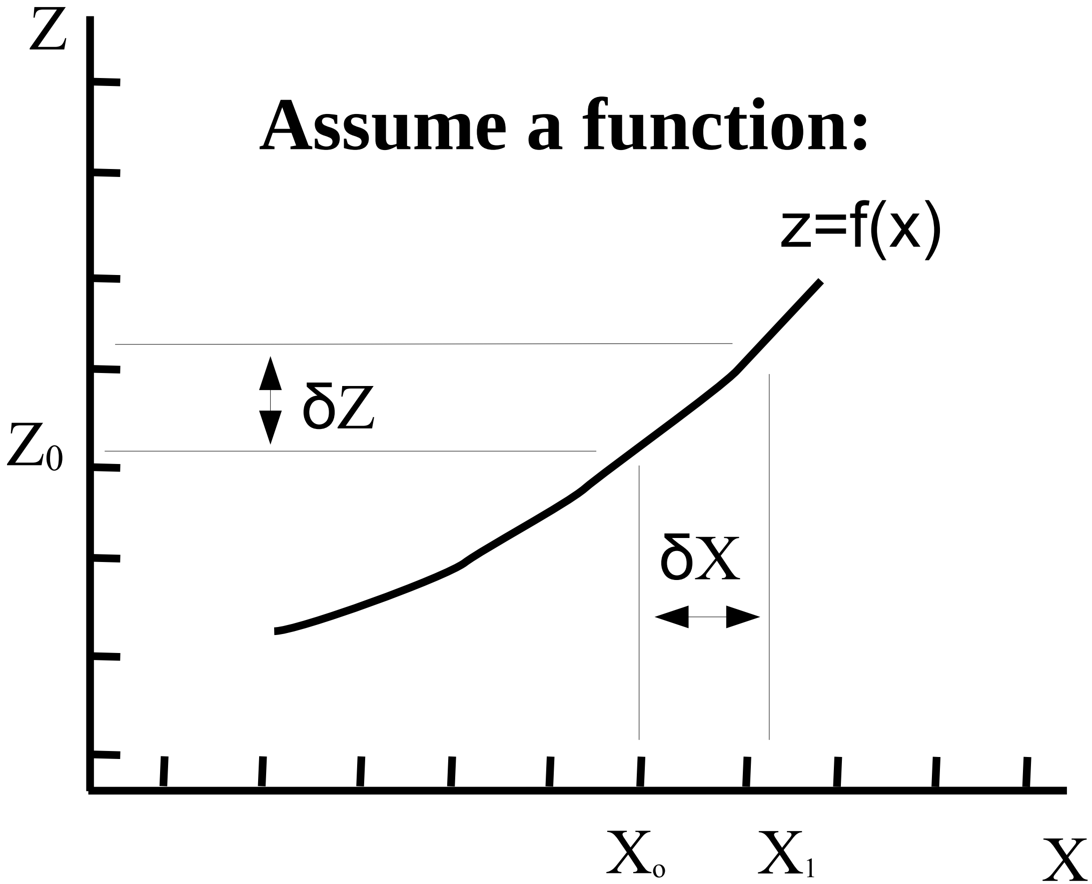
Uncertain and Agreement

- When do two measurements (M) agree?
- When do two measurements (M) agree within one standard deviation of uncertain?



Uncertainty in Function of a Single Variable

Assume a function:



$$\frac{dz}{dx} = \frac{d(f(x))}{dx}$$

$$\delta z = \frac{d(f(x))}{dx} \delta x$$

$\pm \delta x$ - Uncertainty in x

$\pm \delta z$ - Uncertainty in z

Uncertainty in Function of Two Variables

Assume a function: $Z = f(x, y)$

The appropriate quantity for calculating δz is the

Total Differential: $dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$

Treat the differential as a finite difference δz that is calculated from the uncertainties in δx and δy.

$$\delta z = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y$$

Breadth of Distributions

- How reliable is it to use a single number to represent the whole distribution?
- The quantity that is almost universally used to measure the breadth of the distribution is the "standard deviation".
- The "best estimate" of the universe standard deviation is given by:

$$S = \sqrt{\frac{\sum (\bar{\chi} - \chi_i)^2}{N-1}}$$

$\bar{\chi}$ - Arithmetic Average or Mean

χ_i- Value of an Observation

N - number of Observations

Standard Deviation of Calculated Quantities

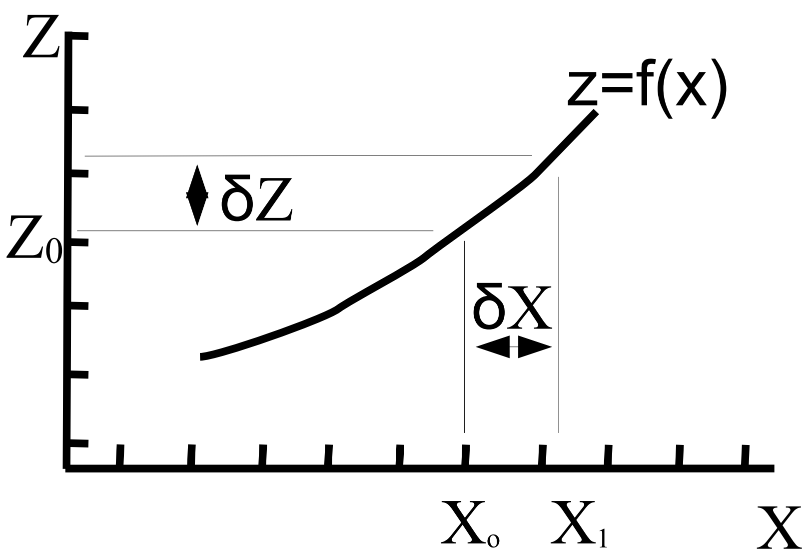
Assume a function: $Z = f(x, y)$

Standard Deviation for the N different z values:

$$S_z = \sqrt{\frac{\sum (\delta z)^2}{N}}$$

Finite Difference δz is:

$$\delta z = \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y$$



$$S_z = \sqrt{\frac{\sum (\delta z)^2}{N}}$$

Standard Deviation of Calculated Quantities

Gives: $S_z^2 = \frac{1}{N} \sum \left(\frac{\partial z}{\partial x} \delta x + \frac{\partial z}{\partial y} \delta y \right)^2$

$$S_z^2 = \frac{1}{N} \sum \left(\left(\frac{\partial z}{\partial x} \right)^2 (\delta x)^2 + \left(\frac{\partial z}{\partial y} \right)^2 (\delta y)^2 + 2 \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \partial x \partial y \right)$$

$$S_z^2 = \left(\frac{\partial z}{\partial x} \right)^2 \frac{1}{N} \sum (\delta x)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \frac{1}{N} \sum (\delta y)^2 + \frac{2}{N} \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \sum (\delta x \delta y)$$

$$S_x^2 = \boxed{\frac{1}{N} \sum (\delta x)^2} \qquad S_y^2 = \boxed{\frac{1}{N} \sum (\delta y)^2} \qquad \boxed{\sum (\delta x \delta y) = 0}$$

$$S_z^2 = \left(\frac{\partial z}{\partial x} \right)^2 \frac{1}{N} \sum (\delta x)^2 + \left(\frac{\partial z}{\partial y} \right)^2 \frac{1}{N} \sum (\delta y)^2 + \frac{2}{N} \frac{\partial z}{\partial x} \frac{\partial z}{\partial y} \sum (\delta x \delta y)$$

Standard Deviation of Calculated Quantities

Gives: $$S_z = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 S_x^2 + \left(\frac{\partial z}{\partial y}\right)^2 S_y^2}$$

How Quickly Functions Changes

Spread of the Uncertainty

- If z is a function of more than two variables, the equation is extended by adding similar terms.

Reference Material

Chapter 2 and 3 of Experimentation: An Introduction to Measurement Theory and Experiment Design by D. C. Baird.